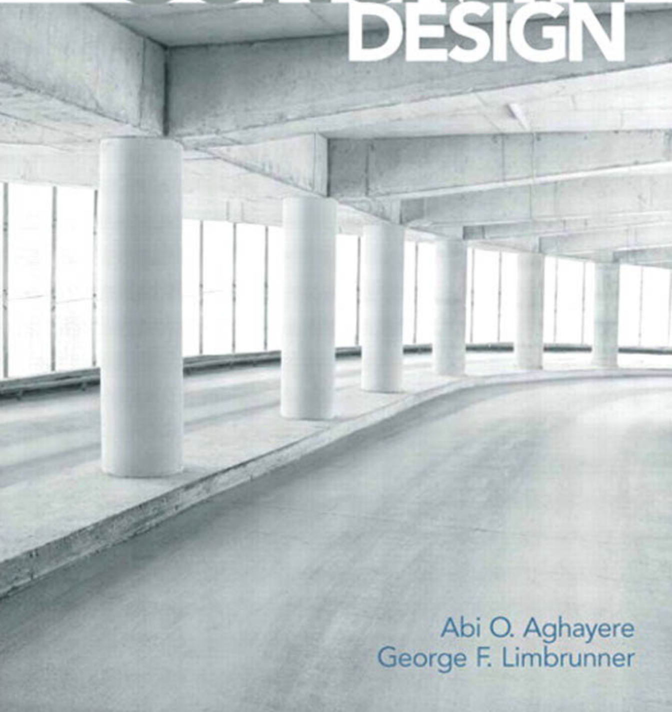
Eighth Edition

REINFORCED CONCRETE DESIGN



Abi O. Aghayere
George F. Limbrunner

Reinforced Concrete Design

EIGHTH EDITION

Abi O. Aghayere, P.Eng.
Drexel University

George F. Limbrunner, PE
Hudson Valley Community College (Emeritus)

PEARSON

Boston Columbus Indianapolis New York San Francisco Upper Saddle River
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montreal Toronto
Delhi Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

Editorial Director: Vernon R. Anthony
Senior Acquisitions Editor: Lindsey Prudhomme Gill
Editorial Assistant: Nancy Kesterson
Director of Marketing: David Gesell
Senior Marketing Coordinator: Alicia Wozniak
Senior Marketing Assistant: Les Roberts
Program Manager: Maren L. Miller
Project Manager: Holly Shufeldt
Art Director: Jayne Conte
Cover Designer: Bruce Kenselaar
Image Permission Coordinator: Mike Lackey
Cover Photo: Shutterstock
Media Director: Karen Bretz
Full-Service Project Management and Composition: Integra Software Services, Pvt. Ltd.
Printer/Binder: Edwards Brothers Malloy
Cover Printer: Lehigh-Phoenix Color

Credits and acknowledgments borrowed from other sources and reproduced, with permission, in this textbook appear on the appropriate page within text.

Library of Congress Cataloging-in-Publication Data

LOC CIP data not available at time of publication.

10 9 8 7 6 5 4 3 2 1

PEARSON

ISBN 10: 0-13-285929-7
ISBN 13: 978-0-13-285929-5

NOTICE TO THE READER

The information contained in this book has been prepared in accordance with recognized engineering principles and is for general information only. Although it is believed to be accurate, this information should not be used for any specific application without competent professional examination and verification of its accuracy, suitability, and applicability by a licensed professional engineer, architect, or designer. The authors and publisher of this book make no warranty of any kind, expressed or implied, with regard to the material contained in this book nor shall they be liable for any special, consequential, or exemplary damages resulting, in whole or in part, from the reader's use of or reliance on this material.

This page intentionally left blank

PREFACE

The primary objective of *Reinforced Concrete Design*, eighth edition, remains the same as that of the previous editions: to provide a basic understanding of the strength and behavior of reinforced concrete members and simple reinforced concrete structural systems.

With relevant reinforced concrete research and literature continuing to become available at a rapid rate, it is the intent of this book to translate this vast amount of information and data into an integrated source that reflects the latest information available. It is not intended to be a comprehensive theoretical treatise of the subject, because it is believed that such a document could easily obscure the fundamentals emphasized in engineering technology and applied engineering programs. In addition, it is believed that adequate comprehensive books on reinforced concrete design do exist for those who seek the theoretical background, the research studies, and more rigorous applications.

This eighth edition has been prepared with the primary objective of updating its contents to conform to the latest *Building Code Requirements for Structural Concrete* (ACI 318-11) of the American Concrete Institute. Because the ACI Code serves as the design standard in the United States, it is strongly recommended that the code be used as a companion publication to this book.

In addition to the necessary changes to conform to the new code, some sections have been edited and a new student design project problem has been added and several drawings updated. Answers to selected problems are furnished at the back of the text.

Throughout the eight editions, the text content has remained primarily a fundamental, non-calculus, and practice-oriented approach to the design and analysis of reinforced concrete structural members using numerous examples and a step-by-step solution format. In addition, there are chapters that provide a conceptual approach on such topics as prestressed concrete and detailing of reinforced concrete structures. The metric system (SI) is introduced in Appendix C with several example problems.

Form design is an important consideration in most structural design problems involving concrete members, and Chapter 12 illustrates procedures for the design of job-built forms for slabs, beams, and columns. Appropriate tables are included that will expedite the design process. In Chapter 14, we introduce practical considerations and rules of thumb for the design of reinforced concrete beams, girders, columns and one way slabs, and methods for strengthening existing reinforced concrete structures.

WHAT'S NEW IN THE EIGHTH EDITION:

- The entire text has been revised to conform to the latest ACI Code: ACI 318-11
- The quadratic equation solution approach for the design of rectangular beams is included in Section 2-14
- A new Chapter 14 that discusses practical considerations and rules of thumb for the design of reinforced concrete structures. Guidance is provided for the initial, preliminary sizing and layout of reinforced concrete structures
- The calculation of approximate moment and shears in concrete girders, which cannot be calculated using the ACI coefficients in Chapter 6, is introduced in Chapter 14
- Repair methods for existing reinforced concrete structures is introduced in Chapter 14
- A student reinforced concrete building design project problem has been added in Chapter 14

This book has been thoroughly tested over the years in engineering technology and applied engineering programs and should serve as a valuable design guide and resource for technologists, technicians, engineering and architectural students, and design engineers. In addition, it will aid engineers and architects preparing for state licensing examinations for professional registration.

As in the past, appreciation is extended to students, past and present, and colleagues who, with their constructive comments, criticisms, and enthusiasm, have provided input and encouragement for this edition.

Download Instructor Resources from the Instructor Resource Center

To access supplementary materials online, instructors need to request an instructor access code. Go to www.pearsonhighered.com/irc to register for an instructor access code. Within 48 hours of registering, you will receive a confirming e-mail including an instructor access code. Once you have received your code, locate your text in the online catalog and click on the Instructor Resources button on the left side of the catalog product page. Select a supplement, and a login page will appear. Once you have logged in, you can access instructor material for all Prentice Hall textbooks. If you have any difficulties accessing the site or downloading a supplement, please contact Customer Service at http://247pearsoned.custhelp.com/

Abi O. Aghayere

George F. Limbrunner

CONTENTS

CHAPTER **ONE**

MATERIALS AND MECHANICS OF BENDING

1-1 CONCRETE

Concrete consists primarily of a mixture of cement and fine and coarse aggregates (sand, gravel, crushed rock, and/or other materials) to which water has been added as a necessary ingredient for the chemical reaction of curing. The bulk of the mixture consists of the fine and coarse aggregates. The resulting concrete strength and durability are a function of the proportions of the mix as well as other factors, such as the concrete placing, finishing, and curing history.

The compressive strength of concrete is relatively high. Yet it is a relatively brittle material, the tensile strength of which is small compared with its compressive strength. Hence steel reinforcing rods (which have high tensile and compressive strength) are used in combination with the concrete; the steel will resist the tension and the concrete the compression. *Reinforced concrete* is the result of this combination of steel and concrete. In many instances, steel and concrete are positioned in members so that they both resist compression.

1-2 THE ACI BUILDING CODE

The design and construction of reinforced concrete buildings is controlled by the *Building Code Requirements for Structural Concrete* (ACI 318-11) of the American Concrete Institute (ACI) [1]. The use of the term *code* in this text refers to the ACI Code unless otherwise stipulated. The code is revised, updated, and reissued on a 3-year cycle. The code itself has no legal status. It has been incorporated into the building codes of almost all states and municipalities throughout the United States, however. When so incorporated, it has official sanction, becomes a legal document, and is part of the law controlling reinforced concrete design and construction in a particular area.

1-3 CEMENT AND WATER

Structural concrete uses, almost exclusively, hydraulic cement. With this cement, water is necessary for the chemical reaction of *hydration*. In the process of hydration, the cement sets and bonds the fresh concrete into one mass. *Portland cement*, which originated in England, is undoubtedly the most common form of cement. Portland cement consists chiefly of calcium and aluminum silicates. The raw materials are limestones, which provide calcium oxide (CaO), and clays or shales, which furnish silicon dioxide (SiO_2) and aluminum oxide (Al_2O_3). Following processing, cement is marketed in bulk or in 94-lb (1-ft^3) bags.

In fresh concrete, the ratio of the amount of water to the amount of cement, by weight, is termed the *water/cement ratio*. This ratio can also be expressed in terms of gallons of water per bag of cement. For complete hydration of the cement in a mix, a water/cement ratio of 0.35 to 0.40 (4 to 4½ gal/bag) is required. To increase the *workability* of the concrete (the ease with which it can be mixed, handled, and placed), higher water/cement ratios are normally used.

1-4 AGGREGATES

In ordinary structural concretes, the aggregates occupy approximately 70% to 75% of the volume of the hardened mass. Gradation of aggregate size to produce close packing is desirable because, in general, the more densely the aggregate can be packed, the better are the strength and durability.

Aggregates are classified as fine or coarse. *Fine aggregate* is generally sand and may be categorized as consisting of particles that will pass a No. 4 sieve (four openings per linear inch). *Coarse aggregate* consists of particles that would be retained on a No. 4 sieve. The maximum size of coarse aggregate in reinforced concrete is governed by various ACI

Code requirements. These requirements are established primarily to ensure that the concrete can be placed with ease into the forms without any danger of jam-up between adjacent bars or between bars and the sides of the forms.

1-5 CONCRETE IN COMPRESSION

The theory and techniques relative to the design and proportioning of concrete mixes, as well as the placing, finishing, and curing of concrete, are outside the scope of this book and are adequately discussed in many other publications [2–5]. Field testing, quality control, and inspection are also adequately covered elsewhere. This is not to imply that these are of less importance in overall concrete construction technology but only to reiterate that the objective of this book is to deal with the design and analysis of reinforced concrete members.

We are concerned primarily with how a reinforced concrete member behaves when subjected to load. It is generally accepted that the behavior of a reinforced concrete member under load depends on the stress–strain relationship of the materials, as well as the type of stress to which it is subjected. With concrete used principally in compression, the compressive stress–strain curve is of primary interest.

The compressive strength of concrete is denoted f'_c and is assigned the units *pounds per square inch* (psi). For calculations, f'_c is frequently used with the units *kips per square inch* (ksi).

A test that has been standardized by the American Society for Testing and Materials (ASTM C39) [6] is used to determine the compressive strength (f'_c) of concrete. The test involves compression loading to failure of a specimen cylinder of concrete. The compressive strength so determined is the highest compressive stress to which the specimen is subjected. Note in Figure 1-1 that f'_c is not the stress that exists in the specimen at failure but that which occurs at a strain of about 0.002. Currently, 28-day concrete strengths (f'_c) range from 2500 to 9000 psi, with 3000 to 4000 psi being common for reinforced concrete structures and 5000 to 6000 psi being common for prestressed concrete members. Concretes of much higher strengths have been achieved under laboratory conditions. The curves shown in Figure 1-1 represent the result of compression tests on 28-day standard cylinders for varying design mixes.

A review of the stress–strain curves for different-strength concretes reveals that the maximum compressive strength is generally achieved at a unit strain of approximately 0.002 in./in. Stress then decreases, accompanied by additional strain. Higher-strength concretes are more brittle and will fracture at a lower maximum strain than will the lower-strength concretes. The initial slope of the curve varies, unlike that of steel, and only approximates a straight line. For steel, where stresses are below the yield point and the material behaves elastically, the stress–strain plot will be a straight line. The slope of the straight line is the modulus of elasticity. For concrete, however, we observe that the straight-line portion of the plot is very short, if it exists at all. Therefore, there exists no constant value of modulus of elasticity for a given concrete because the stress–strain ratio is not constant. It may also be observed that the slope of the initial portion of the curve (if it approximates a straight line) varies with concretes of different strengths. Even if we assume a straight-line portion, the modulus of elasticity is different for concretes of different strengths. At low and moderate stresses (up to about $0.5f'_c$), concrete is commonly assumed to behave elastically.

The ACI Code, Section 8.5.1, provides the accepted empirical expression for *modulus of elasticity*:

$$E_c = w_c^{1.5} 33\sqrt{f'_c}$$

where

E_c = modulus of elasticity of concrete in compression (psi)

w_c = unit weight of concrete (lb/ft^3)

f'_c = compressive strength of concrete (psi)

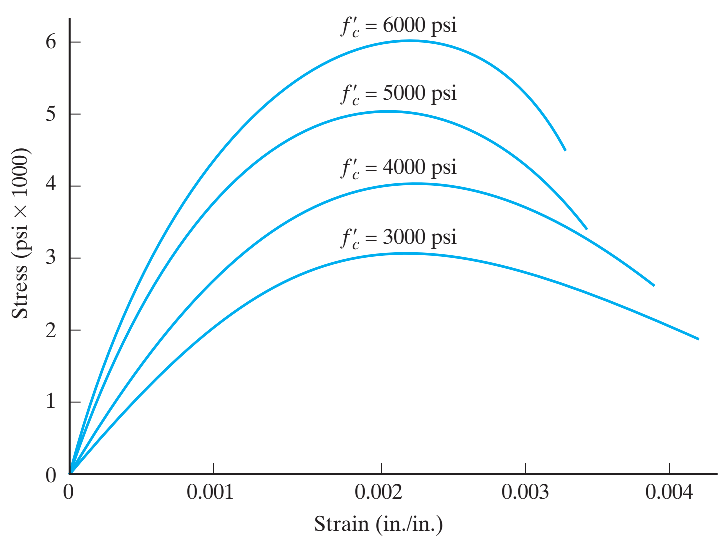


FIGURE 1-1 Typical stress–strain curves for concrete.

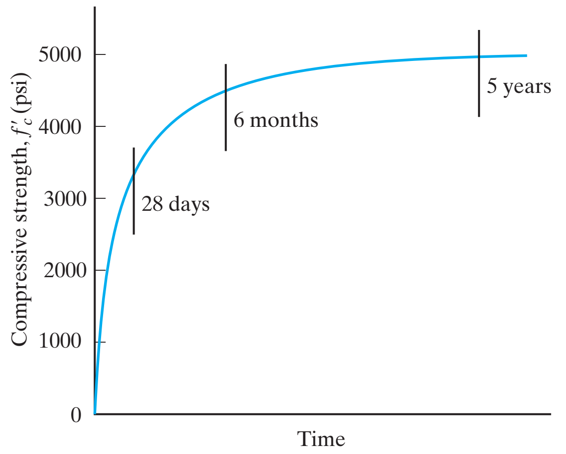


FIGURE 1-2 Strength–time relationship for concrete.

This expression is valid for concretes having w_c between 90 and 160 lb/ft^3. For normal-weight concrete, the unit weight w_c will vary with the mix proportions and with the character and size of the aggregates. If the unit weight is taken as 144 lb/ft^3, the resulting expression for modulus of elasticity is

$$E_c = 57{,}000\sqrt{f'_c} \qquad \text{(see Table A-6 for values of } E_c\text{)}$$

It should also be noted that the stress–strain curve for the same-strength concrete may be of different shapes if the condition of loading varies appreciably. With different *rates of strain* (loading), we will have different-shape curves. Generally, the maximum strength of a given concrete is smaller at slower rates of strain.

Concrete strength varies with time, and the specified concrete strength is usually that strength that occurs 28 days after the placing of concrete. A typical strength–time curve for normal stone concrete is shown in Figure 1-2. Generally, concrete attains approximately 70% of its 28-day strength in 7 days and approximately 85% to 90% in 14 days.

Concrete, under load, exhibits a phenomenon termed *creep*. This is the property by which concrete continues to deform (or strain) over long periods of time while under constant load. Creep occurs at a decreasing rate over a period of time and may cease after several years. Generally, high-strength concretes exhibit less creep than do lower-strength concretes. The magnitude of the creep deformations is proportional to the magnitude of the applied load as well as to the length of time of load application.

1-6 CONCRETE IN TENSION

The tensile and compressive strengths of concrete are not proportional, and an increase in compressive strength is accompanied by an appreciably smaller percentage increase in tensile strength. According to the ACI Code Commentary, the tensile strength of normal-weight concrete in flexure is about 10% to 15% of the compressive strength.

The true tensile strength of concrete is difficult to determine. The *split-cylinder test* (ASTM C496) [6] has been used to determine the tensile strength of lightweight aggregate concrete and is generally accepted as a good measure of the true tensile strength. The split-cylinder test uses a standard 6-in.-diameter, 12-in.-long cylinder placed on its side in a testing machine. A compressive line load is applied uniformly along the length of the cylinder, with support furnished along the full length of the bottom of the cylinder. The compressive load produces a transverse tensile stress, and the cylinder will split in half along a diameter when its tensile strength is reached.

The tensile stress at which splitting occurs is referred to as the *splitting tensile strength*, f_{ct}, and may be calculated by the following expression derived from the theory of elasticity:

$$f_{ct} = \frac{2P}{\pi LD}$$

where

f_{ct} = splitting tensile strength of lightweight aggregate concrete (psi)
P = applied load at splitting (lb)
L = length of cylinder (in.)
D = diameter of cylinder (in.)

Another common approach has been to use the *modulus of rupture*, f_r (which is the maximum tensile bending stress in a plain concrete test beam at failure), as a measure of tensile strength (ASTM C78) [6]. The moment that produces a tensile stress just equal to the modulus of rupture is termed the *cracking moment*, M_{cr}, and may be calculated using methods discussed in Section 1-8. The ACI Code recommends that the modulus of rupture f_r be taken as $7.5\lambda\sqrt{f'_c}$, where f'_c is in psi. Greek lowercase lambda (λ) is a modification factor reflecting the lower tensile strength of lightweight concrete relative to normal-weight concrete. The values for λ are as follows:

Normal-weight concrete—1.0
Sand-lightweight concrete—0.85
All-lightweight concrete—0.75

Interpolation between these values is permitted. See ACI Code Section 8.6.1. for details. If the average splitting tensile strength f_{ct} is specified, then $\lambda = f_{ct}/(6.7\sqrt{f'_c}) \leq 1.0$.

1-7 REINFORCING STEEL

Concrete cannot withstand very much tensile stress without cracking; therefore, tensile reinforcement must be embedded in the concrete to overcome this deficiency. In the United States, this reinforcement is in the form of steel reinforcing bars or welded wire reinforcing composed of steel wire. In addition, reinforcing in the form of structural steel shapes, steel pipe, steel tubing, and high-strength steel tendons is permitted by the ACI Code. Many other approaches have been taken in the search for an economical reinforcement for concrete. Principal among these are the fiber-reinforced concretes, where the reinforcement is obtained through the use of short fibers of steel or other materials, such as fiberglass. For the purpose of this book, our discussion will primarily include steel reinforcing bars and welded wire reinforcing. High-strength steel tendons are used mainly in prestressed concrete construction (see Chapter 11).

The specifications for steel reinforcement published by the ASTM are generally accepted for the steel used in reinforced concrete construction in the United States and are identified in the ACI Code, Section 3.5.

The steel bars used for reinforcing are, almost exclusively, round deformed bars with some form of patterned ribbed projections rolled onto their surfaces. The patterns vary depending on the producer, but all patterns should conform to ASTM specifications. Steel reinforcing bars are readily available in straight lengths of 60 ft. Smaller sizes are also available in coil stock for use in automatic bending machines. The bars vary in designation from No. 3 through No. 11, with two additional bars, No. 14 and No. 18.

For bars No. 3 through No. 8, the designation represents the bar diameter in eighths of an inch. The No. 9, No. 10, and No. 11 bars have diameters that provide areas equal to 1-in.-square bars, 1⅛-in.-square bars, and 1¼-in.-square bars, respectively. The No. 14 and No. 18 bars correspond to 1½-in.-square bars and 2-in.-square bars, respectively, and are commonly available only by special order. Round, plain reinforcing bars are permitted for spirals (lateral reinforcing) in concrete compression members.

ASTM specifications require that identification marks be rolled onto the bar to provide the following information: a letter or symbol indicating the producer's mill, a number indicating the size of the bar, a symbol or letter indicating the type of steel from which the bar was rolled, and for grade 60 bars, either the number 60 or a single continuous longitudinal line (called a *grade line*) through at least five deformation spaces. The *grade* indicates the minimum specified yield stress in ksi. For instance, a grade 60 steel bar has a minimum specified yield stress of 60 ksi. No symbol indicating grade is rolled onto grade 40 or 50 steel bars. Grade 75 bars can have either two grade lines through at least five deformation spaces or the grade mark 75. Reference [7] is an excellent resource covering the various aspects of bar identification.

Reinforcing bars are usually made from newly manufactured steel (billet steel). Steel types and ASTM specification numbers for bars are tabulated in Table A-1. Note that ASTM A615, which is billet steel, is available in grades 40, 60, 75, and 80. Grade 80 steel is allowed for non-seismic applications per ASTM 615 and ASTM 706 [8]. (The full range of bar sizes is not available in grades 40, 75 and 80, however.) Grade 75 steel is approximately 20% stronger than Grade 60 steel requiring a corresponding reduction in the required area of reinforcement, though the installed cost of Grade 75 steel reinforcement is slightly higher than the cost for Grade 60 steel. ASTM A706, low-alloy steel, which was developed to satisfy the requirement for reinforcing bars with controlled tensile properties and controlled chemical composition for weldability, is available in only one grade. Tables A-2 and A-3 contain useful information on cross-sectional areas of bars.

The most useful physical properties of reinforcing steel for reinforced concrete design calculations are yield stress (f_y) and modulus of elasticity. A typical stress–strain diagram for reinforcing steel is shown in Figure 1-3a. The idealized stress–strain diagram of Figure 1-3b is discussed in Chapter 2.

The yield stress (or yield point) of steel is determined through procedures governed by ASTM standards. For practical purposes, the yield stress may be thought of as that stress at which the steel exhibits increasing strain with no increase in stress. The yield stress of the steel will usually be one of the known (or given) quantities in a reinforced concrete design or analysis problem. See Table A-1 for the range of f_y.

The modulus of elasticity of carbon reinforcing steel (the slope of the stress–strain curve in the elastic region) varies over a very small range and has been adopted as 29,000,000 psi (ACI Code, Section 8.5.2).

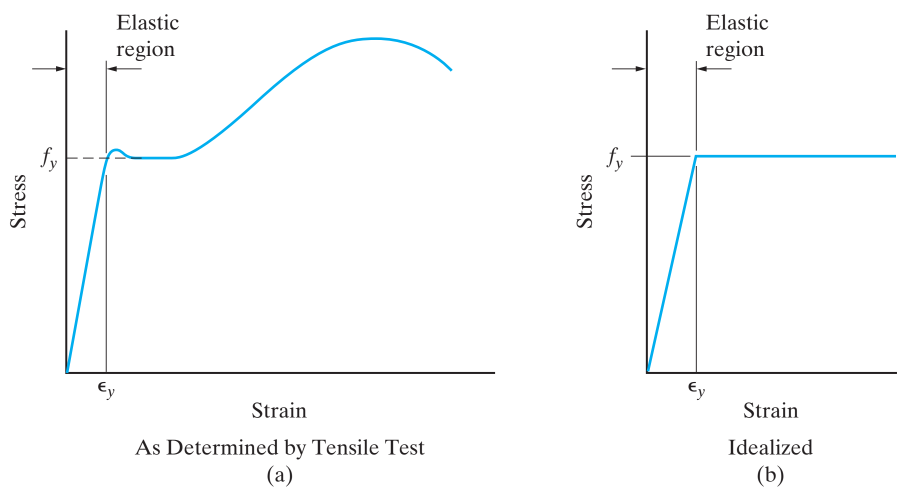


FIGURE 1-3 Stress–strain diagram for reinforcing steel.

Unhindered corrosion of reinforcing steel will lead to cracking and spalling of the concrete in which it is embedded. Quality concrete, under normal conditions, provides good protection against corrosion for steel embedded in the concrete with adequate cover (minimum requirements are discussed in Chapter 2). This protection is attributed to, among other factors, the high alkalinity of the concrete. Where reinforced concrete structures (or parts of structures) are subjected to corrosive conditions, however, some type of corrosion protection system should be used to prevent deterioration. Examples of such structures are bridge decks, parking garage decks, wastewater treatment plants, and industrial and chemical processing facilities.

One method used to minimize the corrosion of the reinforcing steel is to coat the bars with a suitable protective coating. The protective coating can be a nonmetallic material such as epoxy or a metallic material such as zinc (galvanizing). The ACI Code requires epoxy-coated reinforcing bars to comply with ASTM A775 or ASTM A934 and galvanized bars to comply with ASTM A767. The bars to be epoxy coated or zinc coated (galvanized) must meet the code requirements for uncoated bars as tabulated in Table A-1.

Welded wire reinforcing (WWR) (commonly called *mesh*) is another type of reinforcement. It consists of cold-drawn wire in orthogonal patterns, square or rectangular, resistance welded at all intersections. It may be supplied in either rolls or sheets, depending on wire size. WWR with wire diameters larger than about ¼ in. is usually available only in sheets.

Both plain and deformed WWR products are available. Plain WWR must conform to ASTM A185 and be made of wire conforming to ASTM A82. Deformed WWR must conform to ASTM A497 and be made of wire conforming to ASTM A496. Both materials have a yield strength of 70,000 psi. For both materials, the code has assigned a yield strength value of 60,000 psi but makes provision for the use of higher-yield strengths provided the stress corresponds to a strain of 0.35%. The deformed wire is usually more expensive, but it can be expected to have an improved bond with the concrete.

A rational method of designating wire sizes to replace the formerly used gauge system has been adopted by the wire industry. Plain wires are described by the letter W followed by a number equal to 100 times the cross-sectional area of the wire in square inches. Deformed wire sizes are similarly described, but the letter D is used. Thus a W9 wire has an area of 0.090 in.^{2} and a D8 wire has an area of 0.080 in.^{2} A W8 wire has the same cross-sectional area as the D8 but is plain rather than deformed. Sizes between full numbers are given by decimals, such as W9.5.

Generally, the material is indicated by the symbol WWR, followed by spacings first of longitudinal wires, then of transverse wires, and last by the sizes of longitudinal and transverse wires. Thus WWR6 $\times$ 12-W16 $\times$ W8 indicates a plain WWR with 6-in. longitudinal spacing, 12-in. transverse spacing, and a cross-sectional area equal to 0.16 in.^{2} for the longitudinal wires and 0.08 in.^{2} for the transverse wires.

Additional information about WWR, as well as tables relating size number with wire diameter, area, and weight, may be obtained through the Wire Reinforcement Institute [9] or the Concrete Reinforcing Steel Institute [9 and 10]. ACI 318-11 contains a useful chart that gives area (in.^{2}/ft) for various WWR spacings (see Appendix E).

Most concrete is reinforced in some way to resist tensile forces [Figure 1-4]. Some structural elements, particularly

FIGURE 1-4 Concrete construction in progress. Note formwork, reinforcing bars, and pumping of concrete. (George Limbrunner)

footings, are sometimes made of *plain concrete*, however. Plain concrete is defined as structural concrete with no reinforcement or with less reinforcement than the minimum amount specified for reinforced concrete. Plain concrete is discussed further in Chapter 10.

1-8 BEAMS: MECHANICS OF BENDING REVIEW

The concept of bending stresses in homogeneous elastic beams is generally discussed at great length in all strength of materials textbooks and courses. Beams composed of material such as steel or timber are categorized as homogeneous, with each exhibiting elastic behavior up to some limiting point. Within the limits of elastic behavior, the internal bending stress distribution developed at any cross section is linear (straight line), varying from zero at the neutral axis to a maximum at the outer fibers.

The accepted expression for the maximum bending stress in a beam is termed the *flexure formula*,

$$f_b = \frac{Mc}{I}$$

where

f_b = calculated bending stress at the outer fiber of the cross section

M = the applied moment

c = distance from the neutral axis to the outside tension or compression fiber of the beam

I = moment of inertia of the cross section about the neutral axis

The flexure formula represents the relationship between bending stress, bending moment, and the geometric properties of the beam cross section. By rearranging the flexure formula, the maximum moment that may be applied to the beam cross section, called the *resisting moment*, M_R, may be found:

$$M_R = \frac{F_b I}{c}$$

where F_b = the allowable bending stress.

This procedure is straightforward for a beam of known cross section for which the moment of inertia can easily be found. For a reinforced concrete beam, however, the use of the flexure formula presents some complications, because the beam is not homogeneous and concrete does not behave elastically over its full range of strength. As a result, a somewhat different approach that uses the beam's internal bending stress distribution is recommended. This approach is termed the *internal couple method*.

Recall from strength of materials that a couple is a pure moment composed of two equal, opposite, and parallel forces separated by a distance called the *moment arm*, which is commonly denoted Z. In the internal couple method, the couple represents an internal resisting moment and is composed of a compressive force C above the neutral axis (assuming a single-span, simply supported beam that develops compressive stress above the neutral axis) and a parallel internal tensile force T below the neutral axis.

As with all couples, and because the forces acting on any cross section of the beam must be in equilibrium, C must equal T. The internal couple must be equal and opposite to the bending moment at the same location, which is computed from the external loads. It represents a couple developed by the bending action of the beam.

The internal couple method of determining beam strength is more general and may be applied to homogeneous or nonhomogeneous beams having linear (straight-line) or nonlinear stress distributions. For reinforced concrete beams, it has the advantage of using the basic resistance pattern found in the beam.

The following three analysis examples dealing with plain (unreinforced) concrete beams provide an introduction to the internal couple method. Note that the unreinforced beams are considered homogeneous and elastic. This is valid if the moment is small and tensile bending stresses in the concrete are low (less than the tensile bending strength of the concrete) with no cracking of the concrete developing. For this condition, the entire beam cross section carries bending stresses. Therefore, the analysis for bending stresses in the uncracked beam can be based on the properties of the gross cross-sectional area using the elastic-based flexure formula. The use of the flexure formula is valid as long as the maximum tensile stress in the concrete does not exceed the modulus of rupture f_r. If a moment is applied that causes the maximum tensile stress just to reach the modulus of rupture, the cross section will be on the verge of cracking. This moment is called the *cracking moment*, M_{cr}.

These examples use both the internal couple approach and the flexure formula approach so that the results may be compared.

Example 1-1

A normal-weight plain concrete beam is 6 in. × 12 in. in cross section, as shown in Figure 1-5. The beam is simply supported on a span of 4 ft and is subjected to a midspan concentrated load of 4500 lb. Assume f'_c = 3000 psi.

a. Calculate the maximum concrete tensile stress using the internal couple method.
b. Repeat part (a) using the flexure formula approach.
c. Compare the maximum concrete tensile stress with the value for modulus of rupture f_r using the ACI-recommended value based on f'_c.

Solution:

Calculate the weight of the beam (weight per unit length):

$$\text{weight of beam} = \text{volume per unit length} \times \text{unit weight}$$
$$= \frac{6\text{ in.}(12\text{ in.})}{144\text{ in.}^2/\text{ft}^2}(150\text{ lb/ft}^3)$$
$$= 75\text{ lb/ft}$$

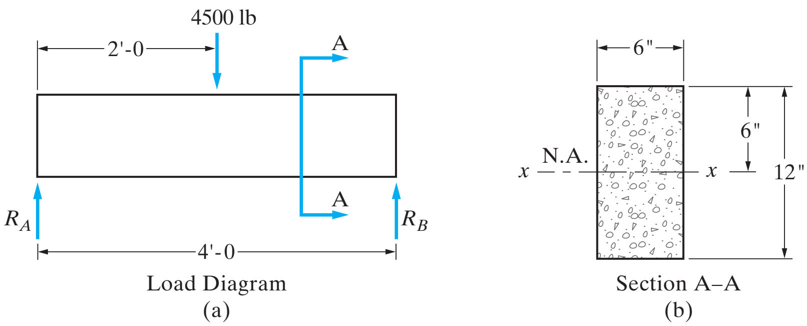


FIGURE 1-5 Sketch for Example 1-1.

Calculate the maximum applied moment:

$$M_{max} = \frac{PL}{4} + \frac{wL^2}{8}$$

$$= \frac{4500 \text{ lb}(4 \text{ ft})}{4} + \frac{75 \text{ lb/ft}(4 \text{ ft})^2}{8}$$

$$= 4650 \text{ ft-lb}$$

a. Internal couple method

1. Because the beam is homogeneous, elastic, and symmetrical with respect to both the X–X and Y–Y axes, the neutral axis (N.A.) is at midheight. Stresses and strains vary linearly from zero at the neutral axis (which is also the centroidal axis) to a maximum at the outer fiber. As the member is subjected to positive moment, the area above the N.A. is stressed in compression and the area below the N.A. is stressed in tension. These stresses result from the bending behavior of the member and are shown in Figure 1-6.
2. C represents the resultant compressive force above the N.A. T represents the resultant tensile force below the N.A. C and T each act at the centroid of their respective triangles of stress distribution. Therefore $Z = 8$ in. C and T must be equal (since $\Sigma H_F = 0$). The two forces act together to form the internal couple (or internal resisting moment) of magnitude CZ or TZ.
3. The internal resisting moment must equal the bending moment due to external loads at any section. Therefore

$$M = CZ = TZ$$

$$4650 \text{ ft-lb } (12 \text{ in./ft}) = C \text{ (8 in.)}$$

from which

$$C = 6975 \text{ lb} = T$$

4. C = average stress × area of beam on which stress acts

$$C = \tfrac{1}{2} f_{top} \text{ (6 in.)(6 in.)} = 6975 \text{ lb}$$

Solving for f_{top} yields

$$f_{top} = 388 \text{ psi} = f_{bott}$$

b. Flexure formula approach

$$I = \frac{bh^3}{12} = \frac{6(12^3)}{12} = 864 \text{ in.}^4$$

$$f_{top} = f_{bott} = \frac{Mc}{I} = \frac{4650(12)(6)}{864} = 388 \text{ psi}$$

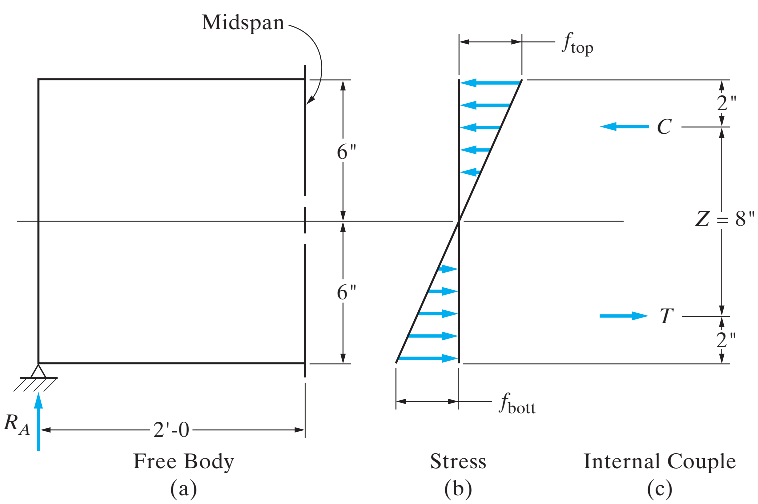


FIGURE 1-6 Sketch for Example 1-1.

c. The ACI-recommended value for the modulus of rupture (based on f_c') is

$$f_r = 7.5\lambda\sqrt{f_c'} = 7.5\,(1.0)\sqrt{3000}$$
$$f_r = 411 \text{ psi}$$

The calculated tensile stress (f_{bott}) of 388 psi is about 6% below the modulus of rupture, the stress at which flexural cracking would be expected.

Example 1-1 is based on elastic theory and assumes the following: (1) a plane section before bending remains a plane section after bending (the variation in strain throughout the depth of the member is linear from zero at the neutral axis), and (2) the modulus of elasticity is constant; therefore, stress is proportional to strain and the stress distribution throughout the depth of the beam is also linear from zero at the neutral axis to a maximum at the outer fibers.

The internal couple approach may also be used to find the moment strength (resisting moment) of a beam.

Example 1-2

Calculate the cracking moment M_{cr} for the plain concrete beam shown in Figure 1-7. Assume normal-weight concrete and $f_c' = 4000$ psi.

a. Use the internal couple method.
b. Check using the flexure formula.

Solution:

The moment that produces a tensile stress just equal to the modulus of rupture f_r is called the cracking moment, M_{cr}. The modulus of rupture for normal-weight concrete is calculated from ACI Equation (9-10):

$$f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{4000} = 474 \text{ psi}$$

For convenience, we will use force units of kips (1 kip = 1000 lb). Therefore, $f_r = 0.474$ ksi.

a. Using the internal couple method

$$Z = 14 - 2(2.33) = 9.34 \text{ in.}$$

$$C = T = \tfrac{1}{2}(0.474)(8)(7) = 13.27 \text{ kips}$$

$$M_{cr} = CZ = TZ = \frac{13.27(9.34)}{12} = 10.33 \text{ ft.-kips}$$

b. Check using the flexure formula

$$f = \frac{Mc}{I}$$

$$M_R = M_{cr} = \frac{f_r I}{c}$$

$$I = \frac{bh^3}{12} = \frac{8(14)^3}{12} = 1829 \text{ in.}^4$$

$$M_{cr} = \frac{f_r I}{c} = \frac{0.474(1829)}{7(12)} = 10.32 \text{ ft.-kips}$$

The internal couple method may also be used to analyze irregularly shaped cross sections, although for homogeneous beams it is more cumbersome than the use of the flexure formula.

Example 1-3

Calculate the cracking moment (resisting moment) for the T-shaped unreinforced concrete beam shown in Figure 1-8. Use $f_c' = 4000$ psi. Assume positive moment (compression in the top). Use the internal couple method and check using the flexure formula.

Solution:

The neutral axis must be located so that the strain and stress diagrams may be defined. The location of the neutral axis with respect to the noted reference axis is calculated from

$$\bar{y} = \frac{\Sigma(Ay)}{\Sigma A} = \frac{4(20)(22) + 5(20)(10)}{4(20) + 5(20)} = 15.33 \text{ in.}$$

The bottom of the cross section is stressed in tension. Note that the stress at the bottom will be numerically larger

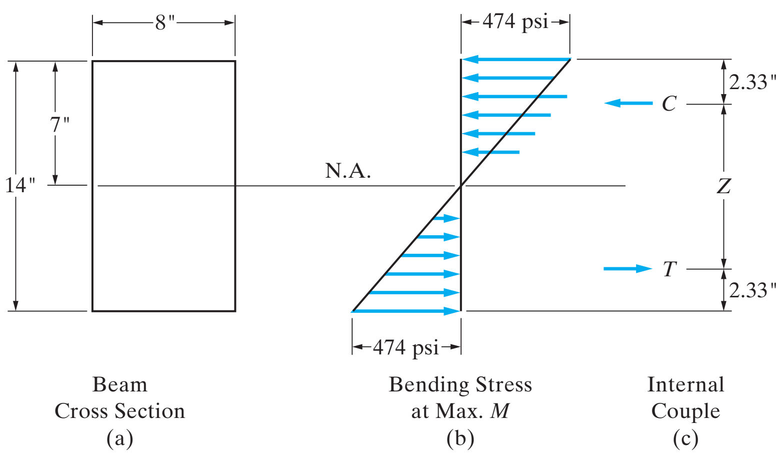


FIGURE 1-7 Sketch for Example 1-2.

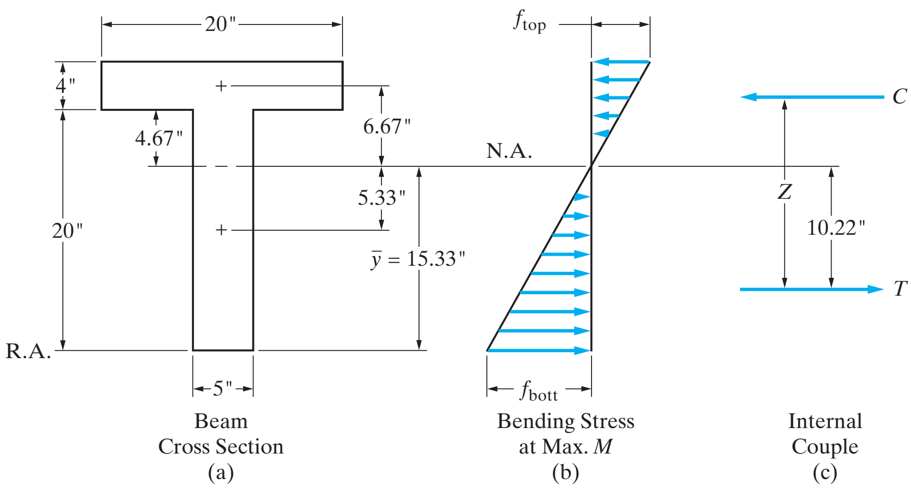


FIGURE 1-8 Sketch for Example 1-3.

than at the top because of the relative distances from the N.A. The stress at the bottom of the cross section will be set equal to the modulus of rupture ($\lambda = 1.0$ for normal-weight concrete):

$$f_{bott} = f_r = 7.5\lambda\sqrt{f'_c} = 7.5(1.0)\sqrt{4000} = 474 \text{ psi} = 0.474 \text{ ksi}$$

Using similar triangles in Figure 1-8b, the stress at the top of the flange is

$$f_{top} = \frac{8.67}{15.33}(0.474) = 0.268 \text{ ksi}$$

Similarly, the stress at the bottom of the flange is

$$f_{\text{bott of flange}} = \frac{4.67}{15.33}(0.474) = 0.1444 \text{ ksi}$$

The total tensile force can be evaluated as follows:

$$\begin{aligned} T &= \text{average stress} \times \text{area} \\ &= \tfrac{1}{2}(0.474)(15.33)(5) = 18.17 \text{ kips} \end{aligned}$$

and its location below the N.A. is calculated from

$$\tfrac{2}{3}(15.33) = 10.22 \text{ in. (below the N.A.)}$$

The compressive force will be broken up into components because of the irregular area, as shown in Figure 1-9. Referring to both Figures 1-8 and 1-9, the component internal compressive forces, component internal couples, and M_R may now be evaluated. The component forces are first calculated:

$$\begin{aligned} C_1 &= 0.1444(20)(4) = 11.55 \text{ kips} \\ C_2 &= \tfrac{1}{2}(0.1236)(20)(4) = 4.94 \text{ kips} \\ C_3 &= \tfrac{1}{2}(0.1444)(5)(4.67) = 1.686 \text{ kips} \\ \text{total } C &= C_1 + C_2 + C_3 = 18.18 \text{ kips} \\ C &\approx T \quad \text{(O.K.)} \end{aligned}$$

Next we calculate the moment arm distance from each component compressive force to the tensile force T:

$$\begin{aligned} Z_1 &= 10.22 + 4.67 + \tfrac{1}{2}(4.00) = 16.89 \text{ in.} \\ Z_2 &= 10.22 + 4.67 + \tfrac{2}{3}(4.00) = 17.56 \text{ in.} \\ Z_3 &= 10.22 + \tfrac{2}{3}(4.67) = 13.33 \text{ in.} \end{aligned}$$

The magnitudes of the component internal couples are then calculated from force $\times$ moment arm as follows:

$$\begin{aligned} M_{R_1} &= 11.55(16.89) = 195.1 \text{ in.-kips} \\ M_{R_2} &= 4.94(17.56) = 86.7 \text{ in.-kips} \\ M_{R_3} &= 1.686(13.33) = 22.5 \text{ in.-kips} \\ M_{cr} &= M_R = M_{R_1} + M_{R_2} + M_{R_3} = 304 \text{ in.-kips} \end{aligned}$$

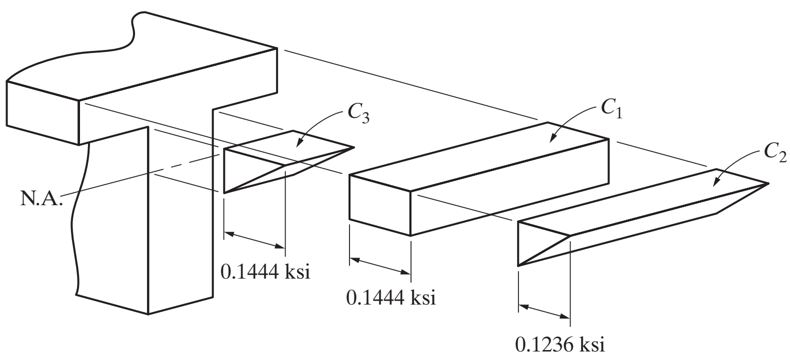


FIGURE 1-9 Component compression forces for Example 1-3.

Check using the flexure formula. The moment of inertia is calculated using the transfer formula from statics:

$$I = \Sigma I_o + \Sigma Ad^2$$

$$I = \tfrac{1}{12}(20)(4^3) + \tfrac{1}{12}(5)(20^3) + 4(20)(6.67^2) + 5(20)(5.33^2)$$
$$= 9840 \text{ in.}^4$$

$$M_{cr} = M_R = \frac{f_r I}{c} = \frac{0.474(9840)}{15.33} = 304 \text{ in.-kips}$$

(Checks O.K.)

As mentioned previously, the three examples are for plain, unreinforced, and uncracked concrete beams that are considered homogeneous and elastic within the bending stress limit of the modulus of rupture. The internal couple method is also applicable to nonhomogeneous beams with nonlinear stress distributions of any shape, however. Because reinforced concrete beams are nonhomogeneous, the flexure formula is not directly applicable. Therefore the basic approach used for reinforced concrete beams is the internal couple method.

References

[1] *Building Code Requirements for Structural Concrete* (ACI 318-11). American Concrete Institute, P.O. Box 9094, Farmington Hills, MI 48333-9094, 2011.

[2] ACI Committee 211. *Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete* (ACI 211.1-91). American Concrete Institute, P.O. Box 9094, Farmington Hills, MI 48333-9094, 1991. (Reapproved 2002.)

[3] George E. Troxell, Harmer E. Davis, and Joe W. Kelly. *Composition and Properties of Concrete*, 2nd ed. New York: McGraw-Hill Book Company, 1968.

[4] *Design and Control of Concrete Mixtures*, 14th ed. Engineering Bulletin of the Portland Cement Association, 5420 Old Orchard Road, Skokie, IL 60077, 2002.

[5] Joseph J. Waddell, ed. *Concrete Construction Handbook*, 3rd ed. New York: McGraw-Hill Book Company, 1993.

[6] *ASTM Standards*. American Society for Testing and Materials, 100 Barr Harbor Drive, West Conshohocken, PA 19428-2959.

[7] *Manual of Standard Practice*, 27th ed. Concrete Reinforcing Steel Institute, 933 North Plum Grove Road, Schaumburg, IL 60173.

[8] Clifford W. Schwinger. "ASTM A615 Grade 75 Reinforcing Steel–When, Why & How to Use It," STRUCTURE Magazine, pp. 34–35, August 2011.

[9] *Manual of Standard Practice*, 7th ed. Wire Reinforcement Institute, 942 Main Street, Suite 300, Hartford, CT 06103, 2006.

[10] Concrete Reinforcing Steel Institute, 933 North Plum Grove Road, Schaumburg, IL 60173.

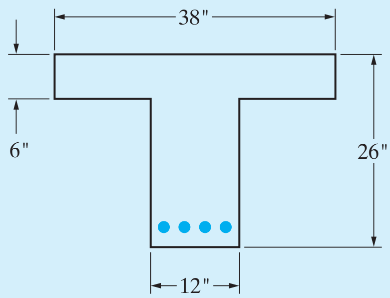
Problems

Note: In the following problems, assume plain concrete to have a weight of 145 pcf (conservative) unless otherwise noted.

1-1. The unit weight of normal-weight reinforced concrete is commonly assumed to be 150 lb/ft^3. Find the weight per lineal foot (lb/ft) for a normal weight reinforced concrete beam that:

a. Has a rectangular cross section 16 in. wide and 28 in. deep.

b. Has a cross section as shown in the accompanying diagram.



PROBLEM 1-1

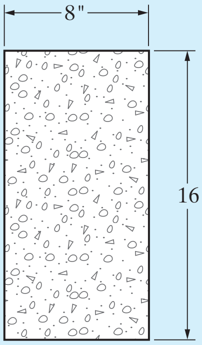
1-2. Develop a spreadsheet application that will display in a table the values of modulus of elasticity E_c for concrete having unit weight ranging from 95 pcf to 155 pcf (in steps of 5 pcf) and compressive strength ranging from 3500 to 7000 psi (in steps of 500 psi). Display the modulus of elasticity rounded to the nearest 1000 psi.

1-3. A normal-weight concrete test beam 6 in. by 6 in. in cross section and supported on a simple span of 24 in. was loaded with a point load at midspan. The beam failed at a load of 2100 lb. Using this information, determine the modulus of rupture f_r of the concrete and compare with the ACI-recommended value based on an assumed concrete strength f'_c of 3000 psi.

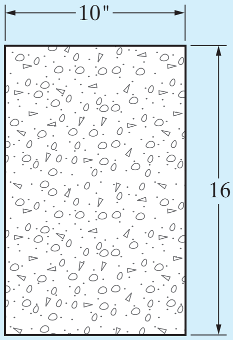
1-4. A plain concrete beam has cross-sectional dimensions of 10 in. by 10 in. The concrete is known to have a modulus of rupture f_r of 350 psi. The beam spans between simple supports. Determine the span length at which this beam will fail due to its own weight. Assume a unit weight of 145 pcf.

1-5. The normal-weight plain concrete beam shown is on a simple span of 10 ft. It carries a dead load (which includes the weight of the beam) of 0.5 kip/ft. There is a concentrated load of 2 kips located at midspan.

Use $f'_c = 4000$ psi. Compute the maximum bending stress. Use the internal couple method and check with the flexure formula.



PROBLEM 1-5



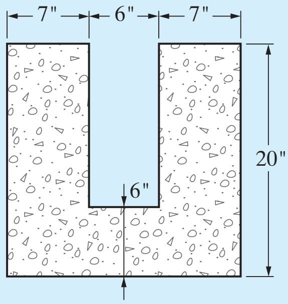
PROBLEM 1-6

1-6. Calculate the cracking moment (resisting moment) for the unreinforced concrete beam shown. Assume normal-weight concrete with $f'_c = 3000$ psi. Use the internal couple method and check with the flexure formula.

1-7. Develop a spreadsheet application to solve Problem 1-6. Set up the spreadsheet so a table will be generated in which the width of the beam varies from 8 in. to 16 in. (1-in. increments) and the depth varies from 12 in. to 24 in. (1-in. increments.) The spreadsheet should allow the user to input any value for f'_c between 3000 and 8000 psi.

1-8. Rework Example 1-3 but invert the beam so that the flange is on the bottom and the web extends vertically upward. Calculate the cracking moment using the internal couple method and check using the flexure formula. Assume positive moment.

1-9. Calculate the cracking moment (resisting moment) for the U-shaped unreinforced concrete beam shown. Assume normal-weight concrete with $f'_c = 3500$ psi. Use the internal couple method and check with the flexure formula. Assume positive moment.

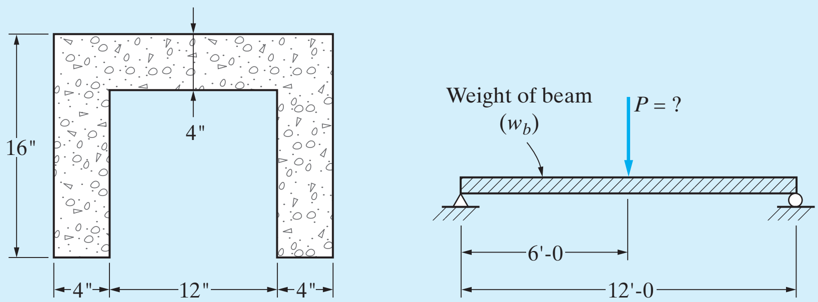


PROBLEM 1-9

1-10. The plain concrete beam shown is used on a 12-ft simple span. The concrete is normal weight with $f'_c = 3000$ psi. Assume positive moment.

a. Calculate the cracking moment.

b. Calculate the value of the concentrated load P at midspan that would cause the concrete beam to crack. (Be sure to include the weight of the beam.)



PROBLEM 1-10

CHAPTER **TWO**

RECTANGULAR REINFORCED CONCRETE BEAMS AND SLABS: TENSION STEEL ONLY

2-1 INTRODUCTION

When a beam is subjected to bending moments (also termed *flexure*), bending strains are produced. Under positive moment (as normally defined), compressive strains are produced in the top of the beam and tensile strains are produced in the bottom. These *strains* produce *stresses* in the beam, compression in the top, and tension in the bottom. Bending members must therefore be able to resist both tensile and compressive stresses.

For a concrete flexural member (beam, wall, slab, and so on) to have any significant load-carrying capacity, its basic inability to resist tensile stresses must be overcome. By embedding reinforcement (usually deformed steel bars) in the tension zones, a *reinforced concrete* member is created. When properly designed and constructed, members composed of these materials perform very adequately when subjected to flexure.

Initially, we will consider simply supported single-span beams that, as they carry only positive moment (tension in the bottom), will be reinforced with steel bars placed near the bottom of the beam.

2-2 ANALYSIS AND DESIGN METHOD

In the beam examples in Chapter 1, we assumed both a straight-line strain distribution and straight-line stress distribution from the neutral axis to the outer fibers. This, in effect, stated that stress was proportional to strain. This analysis is sometimes called *elastic design*.

As stated in Chapter 1, elastic design is considered valid for the homogeneous plain concrete beam as long as the tensile stress does not exceed the modulus of rupture, that stress at which tensile cracking commences. With homogeneous materials used in construction, such as structural steel and timber, the limit of stress–strain proportionality is generally termed the *proportional limit*. Note that the modulus of rupture for the plain concrete beam may be considered analogous to the proportional limit for structural steel and timber with respect to the limit of stress–strain proportionality.

With structural steel, the proportional limit and yield stress have nearly the same value, and when using the allowable stress design (ASD) method, an allowable bending stress is determined by applying a factor of safety to the yield stress.

With timber, the determination of an allowable bending stress is less straightforward, but it may be thought of as some fraction of the breaking bending stress. Using the allowable bending stress and the assumed linear stress–strain relationship, both the analysis and design of timber members and structural steel members (using the ASD method) are performed by a method that is similar to that used in the Chapter 1 examples.

Even though a reinforced concrete beam was known to be a nonhomogeneous member, for many years the elastic behavior approach was considered valid for concrete design, and it was known as the *working stress design* (WSD) method. The basic assumptions for the WSD method were as follows: (1) A plane section before bending remains a plane section after bending; (2) Hooke's law (stress is proportional to strain) applies to both the steel and the concrete; (3) the tensile strength of concrete is zero and the reinforcing steel carries all the tension; and (4) the bond between the concrete and the steel is perfect, so no slip occurs.

Based on these assumptions, the flexure formula was still used even though the beam was nonhomogeneous. This was accomplished by theoretically transforming one material into another based on the ratio of the concrete and steel moduli of elasticity.

Although the WSD method was convenient and was used for many years, it has been replaced with a more modern and realistic approach for the analysis and design of reinforced concrete. One basis for this approach is that at some point in the loading, the proportional stress–strain relationship for the compressive concrete ceases to exist. When first developed, this method was called the *ultimate strength design* (USD) method. Since then, the name has been changed to the *strength design method.*

The assumptions for the strength design method are similar to those itemized for the WSD method, with one notable exception. Research has indicated that the compressive concrete stress is approximately proportional to strain up to only moderate loads. With an increase in load, the approximate proportionality ceases to exist, and the compressive stress diagram takes a shape similar to the concrete compressive stress–strain curve of Figure 1-1. Additional assumptions for strength design are discussed in Section 2-4.

A major difference between ASD and strength design lies in the way the applied loads (i.e., *service loads*—the loads that are specified in the general building code) are handled and in the determination of the capacity (strength) of the reinforced concrete members. In the strength design method, service loads are amplified using load factors. Members are then designed so that their practical strength at failure, which is somewhat less than the true strength at failure, is sufficient to resist the amplified loads. The strength at failure is still commonly called the *ultimate strength*, and the load at or near failure is commonly called the *ultimate load.* The stress pattern assumed for strength design is such that predicted strengths are in substantial agreement with test results.

2-3 BEHAVIOR UNDER LOAD

Before discussing the strength design method, let us review the behavior of a long-span, rectangular reinforced concrete beam as the load on the beam increases from zero to the magnitude that would cause failure. The reinforced concrete simple beam of Figure 2-1 is assumed subjected to downward loading, which will cause positive moment in the beam. Steel reinforcing, three bars in this example, is located near the bottom of the beam, which is the tension side. Note that the overall depth of the beam is designated h, whereas the location of the steel, referenced to the compression face, is defined by the *effective depth, d.* The effective depth is measured to the centroid of the reinforcing steel. In this example, the centroid is at the center of the single layer of bars. If there are multiple layers of bars, then the effective depth is measured from the compression face to the centroid of the bar group.

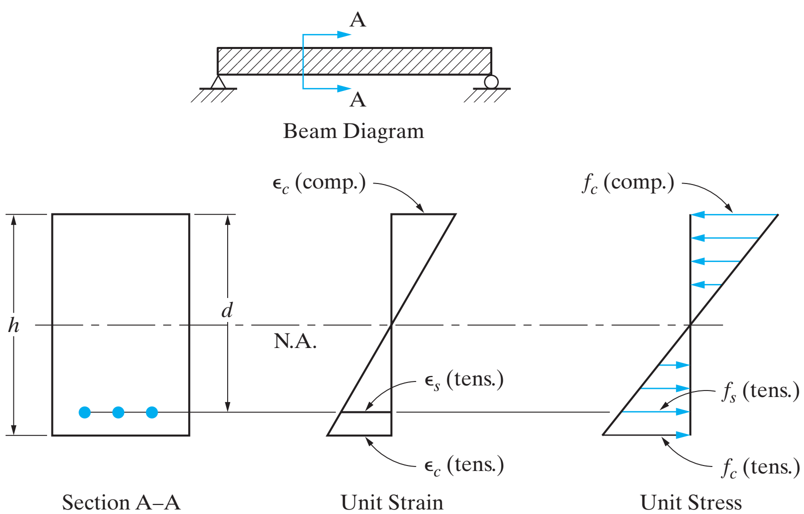


FIGURE 2-1 Flexural behavior at very small loads.

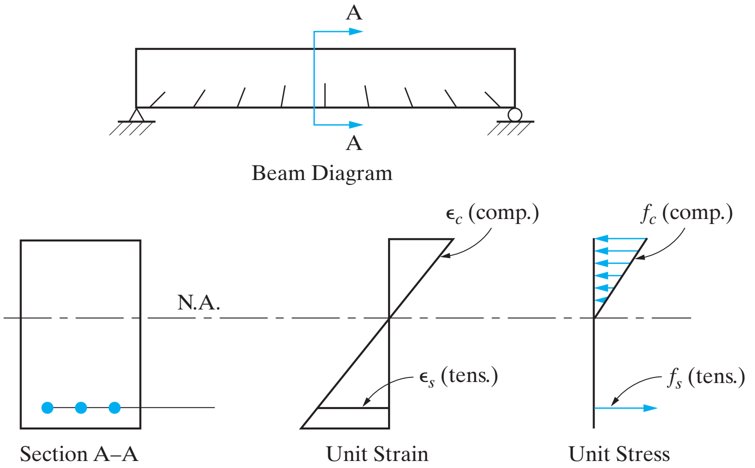


FIGURE 2-2 Flexural behavior at moderate loads.

At very small loads, assuming that the concrete has not cracked, both concrete and steel will resist the tension, and concrete alone will resist the compression. The stress distribution will be as shown in Figure 2-1. The strain variation will be linear from the neutral axis to the outer fiber. Note that stresses also vary linearly from zero at the neutral axis and are, for all practical purposes, proportional to strains. This will be the case when stresses are low (below the modulus of rupture).

At moderate loads, the tensile strength of the concrete will be exceeded, and the concrete will crack (hairline cracks) in the manner shown in Figure 2-2. Because the concrete cannot transmit any tension across a crack, the steel bars will then resist the entire tension. The stress distribution at or near a cracked section then becomes as shown in Figure 2-2. This stress pattern exists up to approximately a concrete stress f_c of about $f'_c/2$. The concrete compressive stress is still assumed to be proportional to the concrete strain.

With further load increase, the compressive strains and stresses will increase; they will cease to be proportional, however, and some nonlinear stress curve will result on the compression side of the beam. This stress curve above the neutral axis will be essentially the same shape as the concrete stress–strain curve (see Figure 1-1). The stress and strain distribution that exists at or near the ultimate load is shown in Figure 2-3. Eventually, the ultimate capacity of the beam will be reached and the beam will fail. The actual mechanism of the failure is discussed later in this chapter.

At this point the reader may well recognize that the process of attaining the ultimate capacity of a member is irreversible. The member has cracked and deflected significantly; the steel has yielded and will not return to its original length. If other members in the structure have similarly reached their ultimate capacities, the structure itself is probably crumbling and in a state of distress or partial ruin, even though it may not have completely collapsed. Naturally, although we cannot ensure that this state will never be reached, factors are introduced to create the commonly accepted margins of safety. Nevertheless the ultimate capacities of members are, at present, the basis for reinforced concrete analysis and design. In this text, it is in such a context that we will speak of failures of members.

2-4 STRENGTH DESIGN METHOD ASSUMPTIONS

The development of the strength design approach depends on the following basic assumptions:

1. A plane section before bending remains a plane section after bending. That is, the strain throughout the depth of the member varies linearly from zero at the neutral axis. Tests have shown this assumption to be essentially correct.
2. Stresses and strains are approximately proportional only up to moderate loads (assuming that the concrete stress does not exceed approximately $f'_c/2$). When the load is increased and approaches an ultimate load, stresses and strains are no longer proportional. Hence the variation in concrete stress is no longer linear.
3. In calculating the ultimate moment capacity of a beam, the tensile strength of the concrete is neglected.

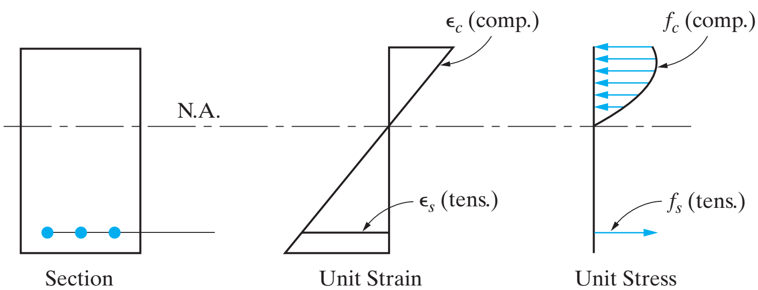


FIGURE 2-3 Flexural behavior near ultimate load.

4. The maximum usable concrete compressive strain at the extreme fiber is assumed equal to 0.003. This value is based on extensive testing, which indicated that the flexural concrete strain at failure for rectangular beams generally ranges from 0.003 to 0.004 in./in. Hence the assumption that the concrete is about to crush when the maximum strain reaches 0.003 is slightly conservative.
5. The steel is assumed to be uniformly strained to the strain that exists at the level of the centroid of the steel. Also, if the strain in the steel (ϵ_s) is less than the yield strain of the steel (ϵ_y), the stress in the steel is $E_s\epsilon_s$. This assumes that for stresses less than f_y, the steel stress is proportional to strain. For strains equal to or greater than ϵ_y, the stress in the reinforcement will be considered independent of strain and equal to f_y. See the idealized stress–strain diagram for steel shown in Figure 1-3b.
6. The bond between the steel and concrete is perfect and no slip occurs.

Assumptions 4 and 5 constitute what may be termed *code criteria* with respect to failure. The true ultimate strength of a member will be somewhat greater than that computed using these assumptions. The strength method of design and analysis of the ACI Code is based on these criteria, however, and consequently so is our basis for bending member design and analysis.

2-5 FLEXURAL STRENGTH OF RECTANGULAR BEAMS

Based on the assumptions previously stated, we can now examine the strains, stresses, and forces that exist in a reinforced concrete beam subjected to its *ultimate moment*, that is, the moment that exists just prior to the failure of the beam. In Figure 2-4, the assumed beam has a width b and an effective depth d and it is reinforced with a steel area of A_s. (A_s is the total *cross-sectional area* of tension steel present.)

Based on the preceding assumptions, it is possible that a beam may be loaded to the point where the maximum tensile steel unit stress equals its yield stress (as a limit) and the concrete compressive strain is less than 0.003 in./in. It is also possible that in another beam, the maximum concrete compressive strain will equal 0.003 in./in. and the tensile steel unit stress will be less than its yield stress f_y. When either condition occurs, it implies a specific mode of failure, which will be discussed later.

As stated previously, the compressive stress distribution above the neutral axis for a flexural member is similar to the concrete compressive stress–strain curve as depicted in Figure 1-1. As may be observed in Figure 2-4, the ultimate compressive stress f'_c does not occur at the outer fiber, neither is the shape of the curve the same for different-strength concretes. Actually, the magnitudes of the compressive concrete stresses are defined by some irregular curve, which could vary not only from concrete to concrete but also from beam to beam. Present theories accept that, at ultimate moment, compressive stresses and strains in concrete are not proportional. Although strains are assumed linear, with maximum strain of 0.003 in./in. at the extreme outer compressive fiber, the maximum concrete compressive stress f'_c develops at some intermediate level near, but not at, the extreme outer fiber.

The flexural strength or resisting moment of a rectangular beam is created by the development of these internal stresses that, in turn, may be represented as internal forces. As observed in Figure 2-4, N_C represents a theoretical internal resultant compressive force that in effect constitutes the total internal compression above the neutral axis. N_T represents a theoretical internal resultant tensile force that in effect constitutes the total internal tension below the neutral axis.

These two forces, which are parallel, equal, and opposite and separated by a distance Z, constitute an internal resisting couple whose maximum value may be termed the *nominal moment strength* of the bending member. As a limit,

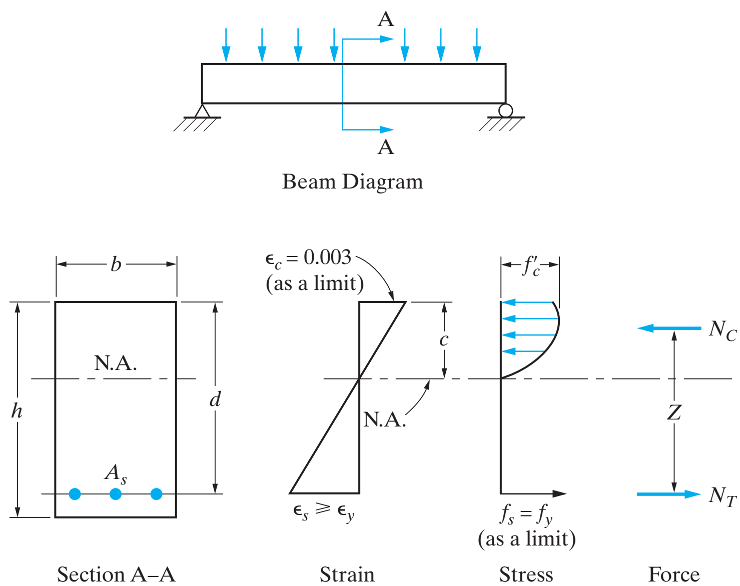


FIGURE 2-4 Beam subjected to ultimate moment.

this nominal moment strength must be capable of resisting the design bending moment induced by the applied loads. Consequently, if we wish to design a beam for a prescribed loading condition, we must arrange its concrete dimensions and the steel reinforcements so that it is capable of developing a moment strength at least equal to the maximum bending moment induced by the loads.

The determination of the moment strength is complex because of the shape of the compressive stress diagram above the neutral axis. Not only is N_C difficult to evaluate but its location relative to the tensile steel is difficult to establish. Because the moment strength is actually a function of the magnitude of N_C and Z, however, it is not really necessary to know the exact shape of the compressive stress distribution above the neutral axis. To determine the moment strength, it is necessary to know only (1) the total resultant compressive force N_C in the concrete and (2) its location from the outer compressive fiber (from which the distance Z may be established). These two values may easily be established by replacing the unknown complex compressive stress distribution by a fictitious one of simple geometrical shape, provided the fictitious distribution results in the same total compressive force N_C applied at the same location as in the actual distribution when it is at the point of failure.

2-6 EQUIVALENT STRESS DISTRIBUTION

For purposes of simplification and practical application, a fictitious but equivalent rectangular concrete stress distribution was proposed by Whitney [1] and subsequently adopted by the ACI Code (Sections 10.2.6 and 10.2.7). The ACI Code also stipulates that other compressive stress distribution shapes may be used, provided results are in substantial agreement with comprehensive test results. Because of the simplicity of the rectangular shape, however, it has become the more widely used fictitious stress distribution for design purposes.

With respect to this equivalent stress distribution as shown in Figure 2-5, the average stress intensity is taken as $0.85\ f'_c$ and is assumed to act over the upper area of the beam cross section defined by the width b and a depth of a. The magnitude of a may be determined by

$$a = \beta_1 c$$

where

c = distance from the outer compressive fiber to the neutral axis

β_1 = a factor that is a function of the strength of the concrete as follows and as shown in Figure 2-6:

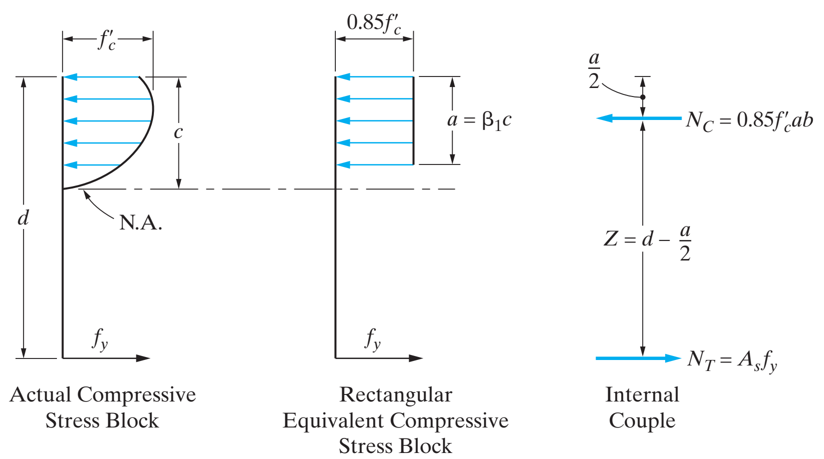


FIGURE 2-5 Equivalent stress block for strength design and analysis.

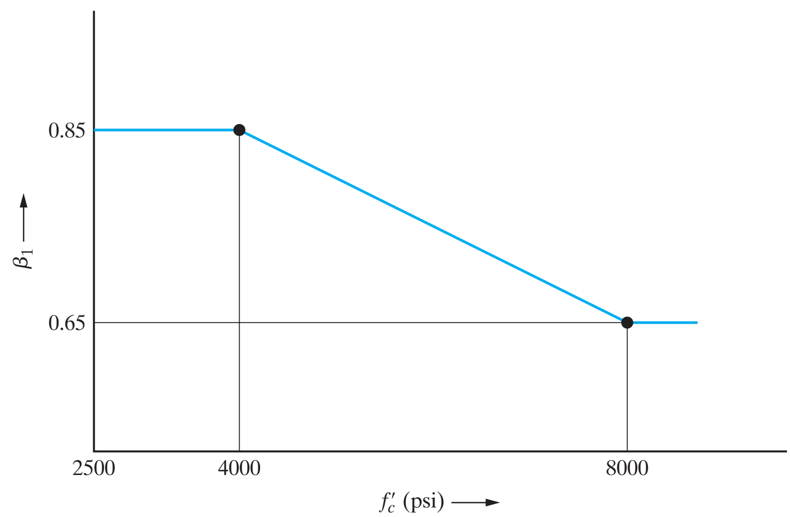


FIGURE 2-6 β_1 v. f'_c.